Modclark Model: Improvement and Application

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Abstract: *- This research is an investigation of a spatially distributed unit hydrograph model. The ModClark model (Peters and Easton, 1997) is an adaptation of Clark's unit hydrograph technique to accommodate gridded NEXRAD precipitation data. In this study, two features were added to the ModClark model: a spatially distributed loss model and a spatially distributed velocity field. A new formula to calculate the spatially distributed velocity field was derived. Maps of spatially distributed runoff curve numbers for Kansas and Oklahoma were developed. The improved ModClark model was applied to 25 storm events on six watersheds. The calibration results are excellent. Two global parameters, the time of concentration and the storage coefficient, were calibrated for each event. Based on the calibration results, two equations to estimate the time of concentration and the storage coefficient were developed. This model and the equations for the two parameters were applied to simulate four storm events on two watersheds. The results are satisfactory.*

Key Words: - *Unit Hydrograph, ModClark Model, GIS, NEXRAD, Spatial Distributed*

I. Introduction

The ModClark spatially distributed hydrograph model was developed by Peters and Easton (1996). It is an adaptation of Clark's unit hydrograph technique to accommodate spatially distributed rainfall data.

The main objectives of this research are to verify the ModClark model, to simulate the spatially varied response of watersheds to spatially varied rainfall, and to provide a framework for applying the model and the NEXRAD precipitation data to ungaged watersheds.

To achieve these objectives, two features were added into the ModClark model: a spatially distributed loss model and a spatially distributed velocity field. A new formula to calculate the spatially distributed velocity field was derived. Maps of spatially distributed runoff curve numbers were developed. The time of concentration, T_C, and the storage coefficient, T_R, are two important parameters for the ModClark model. Based on the calibration results for 25 storm events from six watersheds, two equations to estimate T_C and T_R were developed.

This paper has eight sections. Section 2 presents the development of the ModClark model. Section 3 presents the improved ModClark model. Section 4 presents the preparation of input data for the improved ModClark model. Section 5 presents the application of the improved ModClark model in flood studies. Section 6 presents the calibration of two equations to estimate T_C and T_R. Section 7 presents the application of the improved ModClark model to ungaged watersheds. Section 8 presents conclusion and summary.

II. The Development of the ModClark Model

The Clark model consists of a linear channel in series with a linear reservoir. These two components are modeled separately to account for translation and attenuation. The outflow from the linear channel is the inflow to the linear reservoir and the outflow from the linear reservoir is the IUH. The linear channel component employs an area-time relationship developed by the modeler. Usually, it is assumed that the velocity of flow over the entire area is uniform and the time required for runoff to reach the outlet is directly proportional to the distance. The linear reservoir component of Clark's model represents the lumped effects of storage in the watershed. The outflow from the linear reservoir is computed with a simplified form of the continuity equation. Two parameters, T_C and T_R, are needed to apply the Clark model.

The ModClark model is an adaptation of Clark's unit hydrograph technique to accommodate spatially distributed rainfall data. Its use of radar rainfall for runoff estimation provides a major improvement to the modeling of spatially varied rainfall events (Kull and Feldman, 1998). In this model, the HRAP (Hydrologic

Rainfall Analysis Project) grid is superimposed on the watershed. The net rainfall from each cell is lagged to the watershed outlet and routed through a linear reservoir. The outflows from the linear reservoir are summed and baseflows are added to obtain a total-runoff hydrograph (HEC, 2000). Figure 1 shows the structure of the ModClark model.
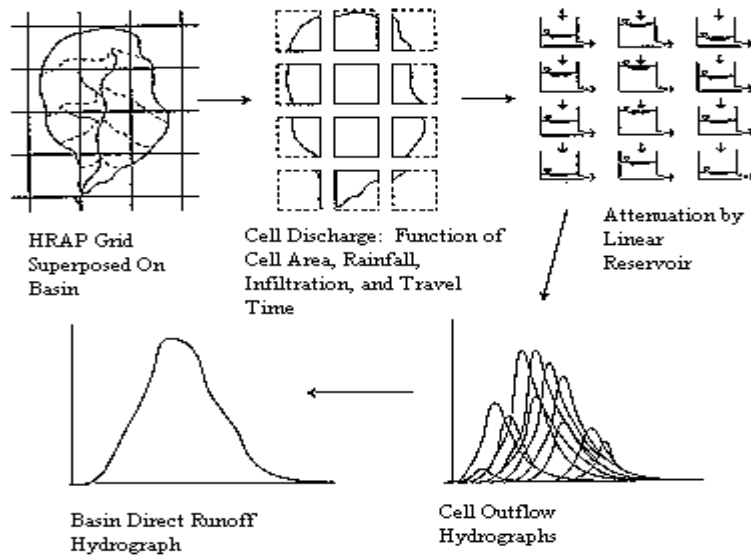
Figure 1: Structure of the ModClark Model (Kull and Feldman, 1998)

The ModClark model requires two global parameters: the time of concentration, T_C, and the storage coefficient (for a linear reservoir), T_R. Both have units of time. Time of concentration, T_C, is used to calculate the translation lag for each path. The flow path from any cell to the outlet consists of a sequence of cells and can be determined from a DEM (Digital Elevation Model). The translation lag for a path is calculated as follows:

$$T_j = \frac{TI_j}{TI_{max}} \cdot T_C \tag{1}$$

where T_j = the travel time for path j
T_C = the time of concentration for a watershed
TI_j = the travel-time index for path j
TI_{max} = the maximum travel-time index in the watershed

Peters and Easton (1996) assumed that the travel velocity is a constant throughout the watershed, so the flow path length can serve as the travel time index.

The lagged net rainfall for each cell is routed through a linear reservoir with the following equation:

$$O_i = \left[\frac{\Delta t}{T_R + 0.5 \cdot \Delta t}\right] \cdot I_{avg} + \left[1 - \frac{\Delta t}{T_R + 0.5 \cdot \Delta t}\right] \cdot O_{i-1} \tag{2}$$

where O_i is the direct runoff at time i, T_R is the storage coefficient, Δt is the time interval, and I_{avg} is the average inflow for the time interval i-1 to i.
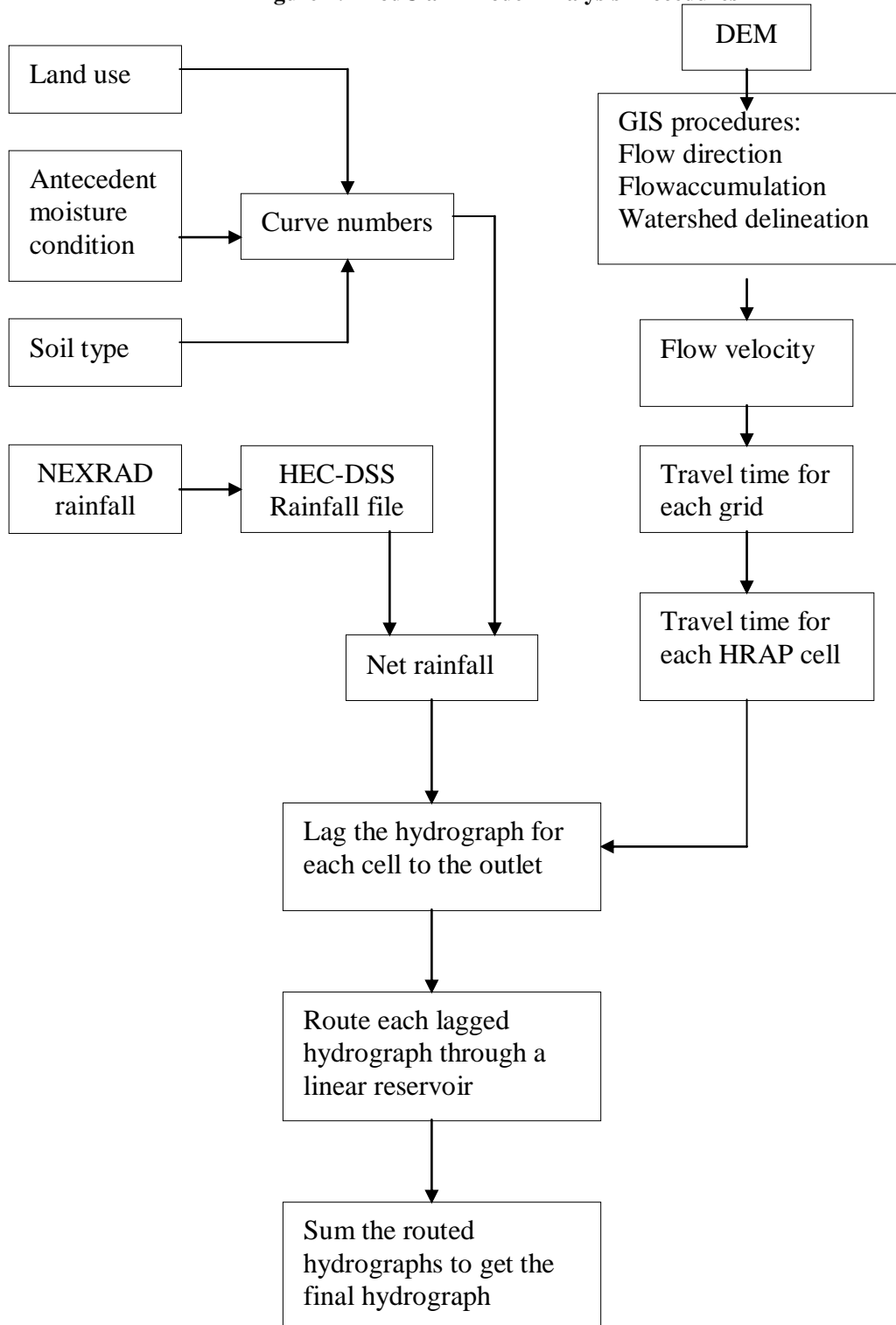
III. The Improved ModClark Model

3.1 Overview of the Improved ModClark Model

Two new features were added to the ModClark model in this study. The ModClark model with these two features is called the improved ModClark model. One new feature is the use of a spatially distributed velocity field instead of constant average velocity. A new travel-time formula has been developed. The other feature is the use of spatially distributed infiltration parameters instead of the basin-averaged infiltration

parameters. The net rainfall is calculated individually for each cell based on both rainfall intensity and loss. Figure 2 shows the flow chart of the improved ModClark model.

Overall, the improved ModClark model takes advantage of the availability of spatially distributed data for rainfall, topography, soils, and land cover. It incorporates spatially distributed rainfall, losses, and velocities while still using the linear hydrologic theory assumed by Clark (1945). This model should better reflect spatially distributed flow characteristics within the watershed.

Figure 2: ModClark Model Analysis Procedures

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  ┌──────────────┐
  │   Land use   │──────────────┐
  └──────────────┘              │
                                ▼
  ┌──────────────┐      ┌───────────────┐        ┌──────────────┐
  │  Antecedent  │      │               │        │     DEM      │
  │  moisture    │────▶ │ Curve numbers │──┐     └──────┬───────┘
  │  condition   │      │               │  │            ▼
  └──────────────┘      └───────────────┘  │   ┌──────────────────────┐
                              ▲            │   │ GIS procedures:       │
  ┌──────────────┐            │            │   │ Flow direction        │
  │  Soil type   │────────────┘            │   │ Flowaccumulation      │
  └──────────────┘                         │   │ Watershed delineation │
                                           │   └──────────┬───────────┘
                                           │              ▼
  ┌──────────────┐      ┌───────────────┐  │   ┌──────────────────┐
  │   NEXRAD     │────▶ │   HEC-DSS     │  │   │  Flow velocity   │
  │   rainfall   │      │ Rainfall file │  │   └────────┬─────────┘
  └──────────────┘      └───────┬───────┘  │            ▼
                                │          │   ┌──────────────────┐
                                │          │   │ Travel time for  │
                                ▼          ▼   │   each grid       │
                           ┌──────────────┐    └────────┬─────────┘
                           │ Net rainfall │             ▼
                           └──────┬───────┘    ┌──────────────────┐
                                  │            │ Travel time for  │
                                  │            │ each HRAP cell    │
                                  │            └────────┬─────────┘
                                  ▼                     │
                    ┌────────────────────────┐          │
                    │ Lag the hydrograph for │◀─────────┘
                    │ each cell to the outlet│
                    └───────────┬────────────┘
                                ▼
                    ┌────────────────────────┐
                    │ Route each lagged      │
                    │ hydrograph through a   │
                    │ linear reservoir       │
                    └───────────┬────────────┘
                                ▼
                    ┌────────────────────────┐
                    │ Sum the routed         │
                    │ hydrographs to get the │
                    │ final hydrograph       │
                    └────────────────────────┘
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3.2 Spatially Distributed Velocity Field and Travel Time Formula

The crucial part of the development of a spatially distributed unit hydrograph model is the determination of the velocity field. Several aspects have to be considered in the selection of a velocity formula. First, a complicated velocity formula will result in complicated calculations. Second, the more factors included in the velocity formula, the more data needed to apply the formula. In practice, the need for too much detailed watershed information will inhibit the use of the model. Third, if the input data are difficult to obtain directly, more assumptions must be made. Too many assumptions will decrease the accuracy of the results. So far, several researchers have proposed different velocity formulas. Most of the proposed formulas require that stream cells and overland-flow cells be distinguished from one another. These limitations are sufficiently restrictive that an alternative approach is desirable.

In this section, new formulas for the velocity and travel time within a grid cell are derived. The derivation performed here is a qualitative analysis. It assumes some simple relationships among the factors.

Leopold (1964) developed the following hydraulic geometry relationships for streams in the midwestern United States:

$$W_a \propto Q^{0.5} \tag{3}$$

$$y_0 \propto Q^{0.4} \tag{4}$$

where W_a is the channel width at bank-full stage, y_0 is the mean channel depth at bank-full stage, and Q is the discharge capacity of the channel at bank-full stage. The hydraulic radius of the channel at bank-full stage, R, can be approximated by the mean depth, which leads to:

$$R \propto Q^{0.4} \tag{5}$$

For a natural stream, the cross-sectional area at bank-full stage can be approximated by $A \propto W_a \cdot y_0 \propto Q^{0.9}$ (6)

or

$$A \propto R^{2.25} \tag{7}$$

From Manning equation,

$$Q \propto \frac{R^{0.67} S^{0.5}}{n} \cdot A \propto \frac{R^{2.92} S^{0.5}}{n} \tag{8}$$

where S is the slope of the energy grade line. The slope of the energy grade line can be approximated by the slope of the grid cell in the direction of flow. Rearranging this equation results in

$$R \propto \left(\frac{nQ}{S^{0.5}}\right)^{0.34} \tag{9}$$

In general, the roughness of a stream tends to decrease slightly in the downstream direction as the bank-full discharge increases. In a study summarized by Barnes (1967), for several streams with characteristics similar to typical Kansas streams, measured peak discharge and Manning n values are as follows:

Q (cfs)	n
65	0.073
1200	0.045
8030	0.038
14500	0.041

From this data set, a relationship between the Manning n and the bank-full discharge can be derived by

$$n \propto Q^{-0.1} \tag{10}$$

Substituting Eq. (9) and Eq. (10) into the Manning equation for the velocity at bank-full flow, we get

$$V \propto \frac{1}{n} R^{0.67} S^{0.5} \propto \frac{1}{Q^{-0.1}} \left(\frac{Q^{-0.1}Q}{C\,S^{0.5}}\right)^{0.23} S^{0.5} \propto Q^{0.31} S^{0.39} \tag{11}$$

Bank-full discharge can be related to watershed characteristics. The USGS has developed regional regression equations for flood discharges with return period from 2 years to 100 years. The bank-full discharge can be approximated by the two-year discharge. Two-year discharge for unregulated rural streams in Kansas can be estimated with the USGS regional regression equation (Rasmussen and Perry, 2000). The regression equation for drainage area over 30 mi^2 is:

$$Q_2 = 0.000182(DA)^{0.532} P^{4.055} \tag{12}$$

Where DA is the drainage area in mi², P is the average annual rainfall for the entire watershed in inches, and Q_2 is the two-year peak in ft³/s. The regression equation for drainage areas under 30 mi² is

$$Q_2 = 0.0126(DA)^{0.579} P^{2.824} \tag{13}$$

Substituting (12) into (11) leads to

$$V \propto S^{0.39} DA^{0.16} P^{1.26} \tag{14}$$

Similarly, for drainage area under 30 mi²,
$$V \propto S^{0.39} DA^{0.18} P^{0.88} \tag{15}$$

A reasonable general approximation is

$$V = K S^{0.4} DA^{0.2} \tag{16}$$

where K is a constant for a watershed. The constant K depends on local rainfall characteristics and can be approximated by

$$K = C P \tag{17}$$

where P is the average annual rainfall.

Even though Eq. (16) is derived from principles of open channel flow, it is also reasonable to apply it to all grid cells along a flow path, including cells with very small drainage areas where overland flow could predominate. The *National Engineering Handbook* of the Natural Resources Conservation Service (1972) recommends the formula of $V = a\,S^{\,b}$ for overland flow velocity, in which S is the land surface slope and a and b are constants. Eq. (16) includes one more factor, upstream drainage area. This factor provides an increase in velocity in the downstream direction. Eq. (16) is used for all cells in the watershed. No distinction is made between stream-channel cells and other cells. Eq. (16) doesn't require detailed information on channel geometry.

The basic idea of the ModClark model is that the net rainfall for each cell is lagged to the watershed outlet by the time of travel from the cell to the watershed outlet. The lagged net rainfall is then routed through a hypothetical linear reservoir and baseflow is added to obtain a total runoff hydrograph. The flow path from any cell to the outlet can be determined from a DEM. The flow path consists of a sequence of cells. By assigning a flow velocity to each cell, the travel time along this flow path can be determined by Eq. (18).

$$T_j = \sum \frac{L_{j,i}}{V_{j,i}} \tag{18}$$

where T_j = the travel time for path j
$V_{j,i}$ = the travel velocity in cell i for path j (from cell j to the watershed outlet)
$L_{j,i}$ = the flow length in cell i for flow path j

Among them the longest travel time (the maximum value of T_j) is the time of concentration, T_C. Substitution of Eq. (16) for $V_{j,i}$ in Eq. (18) leads to:

$$T_j = \frac{1}{K} \sum \frac{L_{j,i}}{(S_{j,i})^{0.40} (DA_{j,i})^{0.20}} \tag{19}$$

In the ModClark model, one must specify the time of concentration for the watershed and a travel-time index for each flow path. The travel-time index for the flow path must be proportional to, but not necessarily equal to, the travel time for the flow path. The travel time for the flow path is computed from the travel-time index and the time of concentration with Eq. (1). The travel time index for path j can be defined as

$$TI_j = KT_j = \sum \frac{L_{j,i}}{(S_{j,i})^{0.40}(DA_{j,i})^{0.20}} \tag{20}$$

The slope, accumulated area, and flow length for each cell can be computed from the DEM of the watershed in Arc/Info. The utility program GridParm from HEC has been modified to perform this calculation and generate the cell parameter file. This file is required for the ModClark model in HEC-HMS.

3.3   Spatially Distributed Curve Numbers
Hydrologic abstractions always vary in space within a watershed. Spatial variations occur because of differences in soil types, land cover and other factors.
Version 2.0 of HEC-HMS incorporates a gridded NRCS curve-number method. For this method, each grid cell is assigned a curve number and the net runoff for each grid cell is computed separately. The gridded curve-number method was used to compute net rainfalls in this study. In this study, maps of runoff curve numbers for Kansas and Oklahoma were derived from land cover and soil data.

IV. Preparation of Input Data for the Improved ModClark Model
Before the ModClark model can be run in HEC-HMS, a radar-based rainfall file in DSS format and a cell parameter file must be generated. Two utility programs, GridLoadNetCDF (HEC, 1995) and GridParm (HEC, 1996), have been written by HEC to create these input files. Radar rainfall data obtained from the ABRFC is stored in the NetCDF (Network Common Data Form) format. The utility program GridLoadNetCDF loads rainfall data into a direct-access file associated with the Hydrologic Engineering Center's Data Storage System (HEC-DSS). The GridParm processes the digital elevation model (DEM) to calculate cell areas and travel time indices. In this study, the GridParm program was modified to incorporate spatially distributed velocity fields and runoff curve numbers.

This section explains the preparation of input data for the improved ModClark model. The major steps are: (1) process the DEM to automatically delineate the watershed and compute the watershed geomorphologic information needed for model input; (2) process the radar-based rainfall data; (3) prepare the input data for the gridded NRCS loss model; and (4) prepare the input data for the exponential recession baseflow model. The Glover River watershed in Oklahoma is used as an example. The drainage area at the gage is 315 mi^2.

4.1 Extracting Morphology Characteristics
This section explains how to determine the three kinds of morphology characteristics needed to develop the velocity field. Watershed delineation procedures are explained in Chapter 4.
The DEM-250K data for the Glover River watershed was downloaded from the USGS web site (http://edcwww.cr.usgs.gov/glis/hyper/guide/1_dgr_demfig/). The spatial resolution of the grid is 100 meters. The DEM data were assembled into a single Arc/Info grid. This grid was transformed from geographic (longitude/latitude) coordinates to the Albers Conic Equal-Area projection.
The development of the velocity field requires three grids: a slope grid, an accumulated area grid, and a flow-length grid. These three grids were generated with the Slope, Flowaccumulation, and Flowlength functions in GRID module of Arc/Info.

• **Accumulated Area**
The Flowaccumulation function creates a grid of accumulated drainage area for each cell. For each cell in the output grid, the result is the number of cells that drain into it. The current cell is not considered in this accumulation. Before the Flowaccumulation command is executed, a flow direction grid must be created with the Flowdirection command. The results of Flowaccumulation can be used to create a stream network by identifying cells with upstream drainage areas above a certain value.

• **Flow Path**
The Flowlength function calculates the length of the longest flow path upstream or downstream of each cell. In this study, the flow length is used to calculate travel time.
Once the slope, accumulated area, and flow path grid have been developed, the travel time index grid can be computed using Eq. (20). The GridParm program was modified to generate the travel-time index grid

and write the values into a cell parameter file. This program also records the HRAP coordinates of each HRAP polygon.

4.2 Radar-Based Rainfall Estimates

Figure 3 shows the HRAP grid superimposed over the Glover River watershed. The watershed areas within the grid cells vary from 0.01 km^2 to 16.80 km^2. Different rainfalls are applied to each HRAP cell. Hourly rainfall data files for the Glover River watershed were downloaded from the ABRFC (http://www.abrfc.noaa.gov/archive/) and unzipped. Each hourly rainfall file was processed with the GridLoadNetCDF program. This program was run with following parameters in DOS:

GridLoadNetCDF grid=hrap b=abrfc x=367 y=263 si=abrfc ds=chik9704 n=05109611z.nc jpg=05109611z.jpg

where GridLoadNetCDF is the program name, b specifies the b part of path name (required by the DSS data set), x and y are the HRAP coordinates of the lower left corner of the Arkansas River Basin, si specifies the watershed name, ds specifies the name of the output DSS file, n specifies the name of input rainfall file, and jpg specifies the name of the output jpeg picture (which can be omitted).

Output data for all of the input rainfall files were written into one DSS output file. A batch file was written to perform these operations.
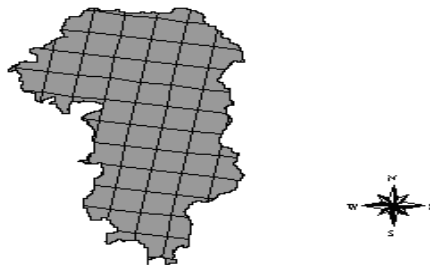
Figure 3: HRAP Grid Superimposed on the Glover River Watershed

4.3 Loss Model

The loss model used in this study is the gridded NRCS curve-number loss model. The NRCS curve number loss model, developed by the NRCS, relates accumulated net rainfall to accumulated rainfall and the runoff curve number. No runoff occurs until the accumulated rainfall exceeds a specified initial abstraction. Thereafter, the accumulated runoff is given by the formulas:

$$P_e = \frac{(P - I_a)^2}{P - I_a + S} \tag{21}$$

and

$$S = \frac{1000}{CN} - 10 \tag{22}$$

where P_e = accumulated runoff (in.)
P = accumulated rainfall (in.)
I_a = initial abstraction (in.)
S = potential maximum retention after runoff begins (in.)
CN = curve number
The initial abstraction is usually approximated by the empirical equation

$$I_a = r \cdot S \tag{23}$$

where r is termed the initial abstraction ratio. The NRCS generally recommends an initial abstraction ratio of 0.2.
Substituting Eq. (23) into Eq. (21) gives

$$P_e = \frac{(P - r \cdot S)^2}{P + (1 - r) \cdot S} \tag{24}$$

Loss and net rainfall were computed independently for each HRAP cell. The NRCS defines three standard antecedent moisture conditions, AMC I, AMC II and AMC III. AMC I represents a condition that is much drier than average, and AMC III represents a condition that is much wetter than average. In HEC-HMS ModClark model, the antecedent moisture condition is quantified by a potential abstraction scale factor, f. This factor is used to adjust the potential maximum abstraction for antecedent conditions. The relationship is

$$S = S_{II} \; ¿ \; f \tag{25}$$

where S_{II} is the maximum potential abstraction for AMC II. S_{II} is computed with Eq. (22) using the curve number for AMC II.

The initial abstraction ratio, r, and the potential abstraction scale factor, f, are two global parameters required by gridded NRCS model. In this study, these two parameters were adjusted manually so that the volume of the simulated runoff hydrograph matched the volume of the observed runoff hydrograph. Some of the manually calibrated values of r and f are out of the ranges recommended by HEC.

4.6 Baseflow Determination

Baseflow is defined as the sustained or fair-weather streamflow. It is composed of groundwater runoff and delayed subsurface runoff (Chow, 1964). In this study, the exponential recession model was adopted. This model relates the baseflow at any time to an initial value as follows:

$$Q_t = Q_0 \; ¿ \; k^t \tag{26}$$

where t is the time since the direct runoff began, Q_t is the baseflow at time t, Q_0 is the initial baseflow at time t = 0, and k is the exponential decay constant. In HEC-HMS, the k value must correspond to t in days. The total streamflow is the sum of the baseflow and the direct surface runoff. The part of the streamflow hydrograph occurring before the recession threshold is reached is computed as the sum of direct runoff and baseflow.

Figure 4 shows the baseflow of selected floods on the Glover River watershed. The baseflow curves for each year are almost parallel, which indicates that different storms have similar k values. The fitted regression equations are shown in Figure 4. A k value was obtained for each storm.

Figure 4: Baseflow Curves and Fitted Regression Equations

4.5 The Input of Observed Streamflow Data

The streamflow data for selected watersheds and flood events in Kansas were obtained from the USGS ADAPS data set. USGS gage streamflow data for Oklahoma was obtained directly from the Oklahoma office of USGS. A C++ program, convert.cpp and a DSS utility program, DSSITS (HEC, 1995), were used to convert the streamflow data to HEC-DSS format.

4.6 HEC-HMS Modeling System

In the HEC-HMS, a project consists of a basin model, a meteorologic model, and control specifications. The basin model is composed of a schematic, a loss model, a transform model, and a baseflow model. The meteorologic model contains the rainfall data. Control specifications set the starting time and date,

the ending time and date, and the computational time interval. The methods used in this study are summarized as follows:

- **Basin Model**
 Loss Method: Gridded NRCS Curve Number Method
Transform Method: ModClark Method
Baseflow Method: Recession Method

- **Meteorologic Model**: Gridded Rainfall Method.
 The cell parameter file required by the ModClark model is imported through the basin model attributes submenu on the basin model screen. The corresponding gridded rainfall DSS file is input through the meteorologic model screen. The user must input a time shift, which is the time difference between UTC time (Coordinated Universal Time, formerly known as Greenwich Mean Time) and local time. The rainfall data files available from ABRFC are in UTC time. UTC is 6 hours ahead of Central Standard Time and 5 hours ahead of Central Daylight Time.
 The optimization module in the HEC-HMS model was used to calibrate two parameters: the time of concentration, T_C, and the storage coefficient, T_R. The objective function used is the discharge-weighted root-mean-square error. The search procedure was the univariate gradient procedure (Hoggan, 1997).

V. The Application of the Improved ModClark Model
In this study, the improved ModClark model was applied to 25 storm events on six watersheds in Kansas and Oklahoma. The time of concentration and the storage coefficient were calibrated for each event. Visual and statistical comparisons were performed, and the errors in peak flow, the times to peak and the runoff volumes were evaluated.

5.1 The Selection of Watersheds
Eight watersheds in Kansas and Oklahoma were selected for this study. All of these streams are unregulated. To evaluate the accuracy of the model at different spatial scales, the drainage areas of selected watersheds ranged from 300 square miles to 900 square miles. Table 1 lists the gage locations and drainage areas. The first six watersheds were used for calibration, and the last two watersheds were used for verification.

Table 1: Watersheds Selected in This Study

USGS Station #	Station Name	Drainage Area (mile^2)	Average Annual Rainfall (in.)	Latitude (ddmmss)	Longitude (ddmmss)
07149000	Medicine Lodge R. near Kiowa, KS	915	26	370217	982804
07151500	Chikaskia R. near Corbin, KS	833	32	370744	973604
07159750	Cottonwood Cr. near Seward, OK	317	33	354849	972840
07191000	Big Cabin Cr. near Big Cabin, OK	428	41	363406	950907
07197000	Baron Fork at Eldon, OK	325	44	355516	945018
07337900	Glover R. near Glover, OK	315	49	340551	945407
07147070	Whitewater R. at Towanda, KS	415	32	374745	970125
07196500	Illinois R. near Tahlequah, OK	929	43	355522	945524

5.2 The Selection of Storm Events
The storm events included in this study have significant rainfall amounts, and the observed hydrographs have significant peak flow. Table 2 lists the areal average rainfalls and peaks for all the events. Table 3 lists all the parameters for loss model and the baseflow model. The parameters for loss model were determined by preliminary calibration of runoff volume. The parameters for the baseflow model were determined from gage data.

Table 2: Areal Averaged Rainfall and Peak Flow for Each Event

Storm #	Rainfall (in)	Peak Flow (cfs)
Medicine River (07149000)		
1	0.65	4890
2	0.52	5020
3	0.74	5130
4	0.90	10300
Chikaskia River (07151500)		
1	0.78	4520
2	1.15	16500
3	0.97	11200
4	1.02	15200
Cottonwood Creek (07159750)		
1	0.75	4452
2	1.34	10552
3	2.09	11500
4	0.64	3594
Cabin Creek (07191000)		
1	1.00	11714
2	1.06	9052
3	1.47	10459
4	2.54	12044
5	1.51	13552
Baron Creek (07197000)		
1	1.97	15165
2	2.36	31552
3	3.91	31535
4	1.40	7669
Glover River (07337900)		
1	3.75	35570
2	4.15	47945
3	5.32	18459
4	5.67	30035

Table 3: Input Parameters for Each Event

| Storm #	Initial Abstraction Ratio	Potential Abstraction Factor	Baseflow Decay Constant	Threshold Value (cfs)	Initial Flow (cfs)
Medicine River (07149000)					
1	0.20	1.50	0.80	800	350
2	0.30	2.28	0.80	1000	300
3	0.20	2.28	0.80	1000	400
4	0.20	2.28	0.80	2000	100
Chikaskia River (07151500)					
1	0.25	2.28	0.70	450	200
2	0.20	2.28	0.70	1650	100
3	0.20	2.28	0.70	1120	230
4	0.20	0.80	0.70	1520	350
Cottonwood Creek (07159750)					
1	0.45	2.28	0.75	500	70
2	0.20	1.00	0.75	770	90
3	0.40	0.30	0.75	200	120
4	0.45	2.28	0.75	300	110
Cabin Creek (07191000)					
1	0.20	0.80	0.60	500	50
2	0.20	0.20	0.60	500	30
3	0.10	0.30	0.60	900	80
4	0.10	0.10	0.60	400	40
5	0.20	0.10	0.60	400	40
Baron Creek (07197000)					
1	0.20	0.20	0.50	5000	200
2	0.10	0.10	0.50	5000	190
3	0.10	0.10	0.50	5000	370
4	0.20	0.40	0.50	3000	500
Glover River (07337900)					
1	0.20	0.43	0.50	5000	340
2	0.20	0.20	0.50	5000	360
3	0.20	1.80	0.50	5000	170
4	0.20	0.25	0.50	8000	360

5.3 The Evaluation of Calibration Results

For flood hydrograph modeling, both visual and statistical comparisons between the simulated and observed hydrographs are recommended (ASCE, 1993). Visual comparisons of simulated and observed hydrographs can provide an overall view of the model performance and a feeling for the model capabilities. For each event, the relative errors in the peak flow, the time to peak flow and the runoff volume were evaluated. The results are summarized in Table 4. Graphs for the 25 storm events are presented in Appendix C. Four storm events from the Glover River watershed are analyzed here.

- **Overview of Selected Calibration Results**

Storm #1 (September 18 –23, 1996)

The spatially averaged rainfall for this event was 4.97 inches. The rainfall occurred in two periods separated by an interval of 6 hours. The main peak flow is caused by the first period of rainfall. The second peak on the falling limb is caused by the second period of rainfall. The falling limb of the observed hydrograph is smooth. The calibrated hydrograph closely matches the observed hydrograph in the shape, the peak flow, the time to peak flow, and the runoff volume. The relative error in the main peak flow is about 1%. The relative error in the second peak flow is 13%. The error in time to peak is one hour. The relative error in runoff volume is 3%.
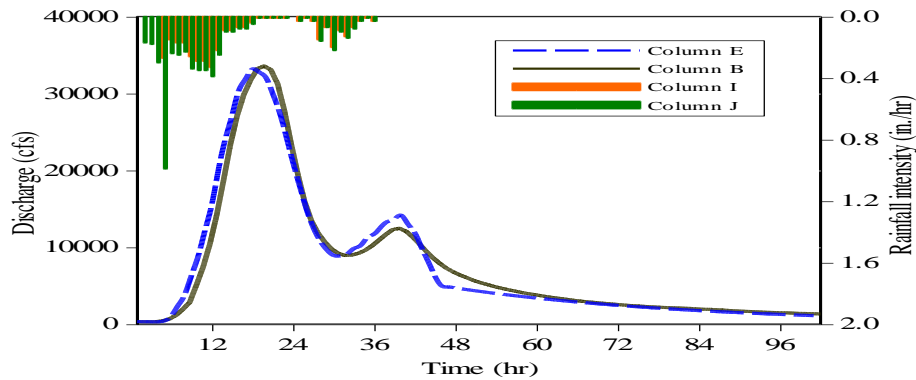
Figure 5 (a): Observed and Calibrated Hydrograph for Storm #1
(November, 24 – 28, 1996)

Storm #2 (February 19-24, 1997)

Storm #2 had a total rainfall of 4.38 inches in two periods. About one third of the rainfall occurred in the first period. The pause between the two periods of rainfall was about 8 hours. The observed hydrograph shows that most of the rainfall in the first period was consumed by the initial abstraction. The peak flow was produced by the second period of rainfall. The calibrated hydrograph shows excellent agreement with the observed hydrograph. No error in time to peak is observed. The shape matches the observed hydrograph well. The relative error in the peak flow is 2.5%. The relative error in the runoff volume is 2%.
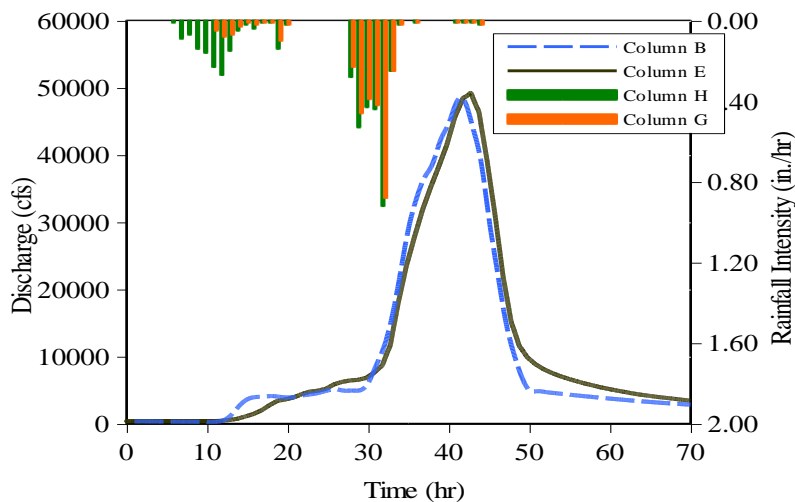
Figure 5 (b): Observed and Calibrated Hydrograph for Storm #2
(February 19-24, 1997)

Storm #3 (December 20-25, 1997)

Storm #3 had a total rainfall of 5.26 inches. The distribution of total rainfall was such that the third rainfall period followed two rainfall periods with a pause of about 48 hours. The first two rainfall periods were mainly consumed by the initial abstraction. They also produced two small peaks. The main peak followed the third rainfall period. The calibrated hydrograph closely matches the observed hydrograph in the main peak flow, the time to peak flow, and the shape. The relative error in the main peak is about 1% and the difference in the times to peak is one hour. For the two small peaks on the rising limb, the model shows too much attenuation. The relative error in the peak discharge is about 47%. The relative error in the runoff volume is 8%.
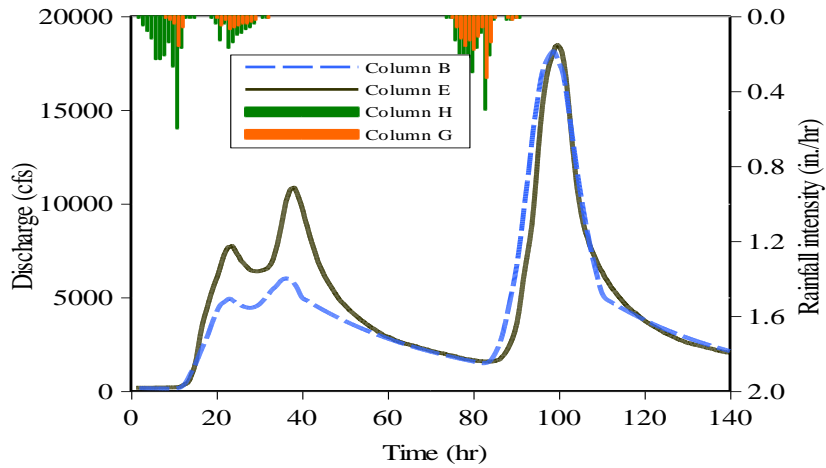
Figure 5 (c): Observed and Calibrated Hydrograph for Storm #3
(December 20 -25, 1997)

Storm #4 (January 04 - 05, 1998)

Storm #4 had a total rainfall of 2.82 inches. There was a long period of light rainfall before the main period of rainfall. The prolonged light rainfall was totally consumed by the initial abstraction and did not generate any runoff. The calibrated hydrograph underestimates the peak discharge by 9%, but it matches the observed hydrograph well in the shape and the time to peak flow. The relative error in runoff volume is 9%.
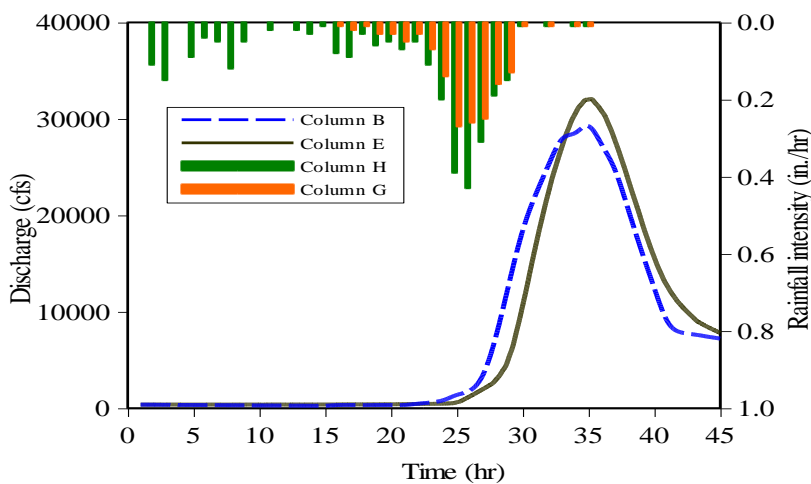
Figure 5 (d): Observed and Calibrated Hydrograph for Storm #4
(January 04 - 05, 1998)

- **Statistical Comparison**

Table 4 summarizes the calibration results for all 25 storm events.

Table 4: Calibration Results for the 25 Storm Events

Storm #	T_c (hr)	T_s (hr)	Relative Error in Peak (%)	Error in Time to Peak (hr)	Relative Error in Volume (%)
Medicine River (07149000)					
1	42	30	1.2	0	11
2	42	28	3.3	3	21
3	42	35	1.4	3	9
4	40	35	2.3	0	15
Chikaskia River (07151500)					
1	25	30	0.0	4	8
2	26	20	0.4	1	6
3	25	16	3.7	2	5
4	27	19	3.2	5	10
Cottonwood Creek (07159750)					
1	26	13	4.2	1	6
2	30	7	0.5	1	8
3	24	5	5.0	1	12
4	30	13	1.1	2	6
Cabin Creek (07191000)					
1	20	12	0.8	1	13
2	20	10	3.4	0	4
3	23	18	5.2	2	7
4	21	14	0.4	2	10
5	22	18	0.5	1	15
Baron Creek (07197000)					
1	24	2	4.8	0	13
2	20	2	4.4	1	15
3	22	2	3.8	2	20
4	25	6	5.1	1	15
Glover River (07337900)					
1	15	3	0.7	1	3
2	16	2	2.5	0	2
3	14	4	0.7	1	8
4	16	2	9.1	0	9

Overall, the improved ModClark model did an excellent job with calibration. Comparing the calibrated hydrograph and the observed hydrograph on an ordinate-by-ordinate basis, the calibrated hydrographs match the observed hydrographs well in the shape, and no obvious bias was observed. Regarding the peak flow, the largest error is about 9%, while most the errors are smaller than 5%. The calibrated models also did a good job in matching the observed times to peak and the observed runoff volumes. The results show that the model can be calibrated satisfactorily for the watersheds as small as 300 square miles and watersheds as large as 900 square miles.

VI. The Estimation of Parameters

The calibrated times of concentration and storage coefficients are fairly consistent for different storms on the same watershed. The average of the values for the different storms is considered the true values for the watershed. Table 5 lists the average values for each watershed.

Table 5: Calibrated Parameters for Selected Watersheds

USGS Station #	T_C (hr)	T_R (hr)
07149000	25.8	32.5
07151500	41.5	21.3
07159750	15.3	10.3
07191000	22.8	14.3
07197000	21.2	3.0
07337900	27.5	3.2

6.1 The Estimation of Time of Concentration

Once the time of concentration has been determined, the value of C in the equation (17) can be computed as follows

$$C = \frac{TI_{max}}{P\, T_c} \qquad (27)$$

where TI_{max} is the largest of the TI values from Eq. (20) for the flow paths in the watershed.

Table 6 lists the values of TI_{max} and C for each watershed (based on P in inches).

Table 6: Maximum Time Indices and C Values for Selected Watersheds

USGS Station #	TI_{max}	C
07149000	14966	13
07151500	10854	11
07159750	8288	13
07191000	9169	7
07197000	6092	10
07337900	6683	6

The C values have a mean of 10 and standard deviation of 1. Substituting the average C value into Eq. (27) leads to

$$T_c = \frac{1}{10\,P} TI_{max} \qquad (28)$$

where P is the average annual rainfall in inches. TI_{max} is calculated from Eq. (20) for L in meter, DA in m^2 and S in m/m percent.

6.2 The Estimation of Storage Coefficient

Several investigators have proposed formulas for estimating the storage coefficient from watershed characteristics. Clark (1945) proposed a formula to estimate the storage coefficient. The proposed formula is

$$T_R = \frac{c\, L}{\sqrt{S}} \qquad (29)$$

where T_R is the storage coefficient in hours, L is the length of the main stream in miles, S is the mean channel slope, and c is a coefficient that varies from 0.8 to 2.2.

Linsley (1949) suggested the formula

$$T_R = \frac{b\, L\, \sqrt{A}}{\sqrt{S}} \qquad (30)$$

where T_R is the storage coefficients in the unit of hours, A is the drainage area in mi², L is the channel length in miles, and b is a coefficient that varies from about 0.04 to 0.08.

Johnstone and Cross (1949) proposed:

$$T_R = b + c\frac{A}{LS} \tag{31}$$

where S is the average overland slope. b and c are experimentally determined constants.

Many regression equations for T_R have also been developed. These formulas have some common features. They all include a variable, either drainage area or channel length, to measure the watershed size and a variable, either channel slope or overland slope, to measure watershed relief. This study seeks to relate T_R to six watershed characteristics using a regression analysis. Six characteristics are defined as follows:

Drainage area (A).
Channel length (L): the total length of the main channel.
Average watershed slope (S): the sum of slopes of all the cells divided by the total number of cells in a watershed.
Watershed shape factor (Sh): the dimensionless ratio L^2/A.
Latitude (Lat): the latitude at the gage.
Longitude (Long): the longitude at the gage.

Table 7 lists the values of these characteristics for the six gaged watersheds.

Table 7: Physical Characteristics of Gaged Watersheds

USGS Station #	Area (mi²)	Average Watershed Slope (%)	Channel Length (miles)	Shape Factor	Latitude (ddmmss)	Longitude (ddmmss)
07149000	915	1.932	108.1	12.9	37.0381	98.0011
07151500	833	1.695	90.9	10.4	37.1289	97.6011
07159750	317	1.215	30.5	2.9	35.8136	97.4778
07191000	428	1.373	36.7	3.1	36.5683	95.1519
07197000	325	4.026	36.6	4.1	35.9211	94.8383
07337900	315	4.220	41.6	5.5	34.0875	94.9019

Table 8: Correlation Matrix

	A	S	L	Sh	Lat	Long
A	1					
S	0.41	1				
L	0.98	0.25	1	1		
Sh	0.94	0.13	0.98	1		
Lat	0.74	0.69	0.62	0.51	1	
Long	0.32	0.53	0.45	0.37	0.36	1

To examine the relationship among these independent variables, correlation analyses were performed. Table 8 shows the correlation matrix. The correlation analysis shows that the latitudes and longitudes are not highly related with any other variables. The average watershed slope is not highly related with any other variables. The drainage area is highly related with channel length (r = 0.98) and shape factor (r = 0.94).

The resulted best-fit regression equation from stepwise analysis is:

$$T_R = 0.0060 \frac{A^{1.33}}{S^{0.98}} \tag{32}$$

where T_R is the storage coefficient in hours, A is the drainage area in mi^2, S is the averaged watershed slope in percent. For this equation, the coefficient of determination (r^2) is 0.98 and the standard error is 0.19.

VII. Application of the Improved ModClark Model to Ungaged Watersheds

Verification of the model and the equations for T_C and T_R are necessary. Four storm events from Whitewater River watershed at Towanda, Kansas (USGS gage # 07147070) and Illinois River watershed near Tahlequah, Oklahoma (USGS gage # 07196500) were selected for this purpose. Times of concentration and storage coefficients were calculated with Eq. (28) and Eq. (32). Constant baseflows were assumed. The initial abstraction ratio was assumed to equal 0.2, a value recommended by the NRCS. The potential abstraction factor was set to 2.28 (AMC I) since no rainfall had occurred in the previous 10 days for all of these four events. Table 9 summarized the parameters used in predication. The results of the verification runs are presented in Figure 6 and Table 10.

Table 9: Parameters for Verification Storm Events

Storm #	T_C (hr)	T_R (hr)	Initial Abstraction Ratio	Potential Abstraction Factor
Whitewater River (07147070)				
1	24	34	0.2	2.28
2	24	34	0.2	2.28
Illinois River (07196500)				
3	37	20	0.2	2.28
4	37	20	0.2	2.28

Table 10: Simulation Results for Verification Storm Events

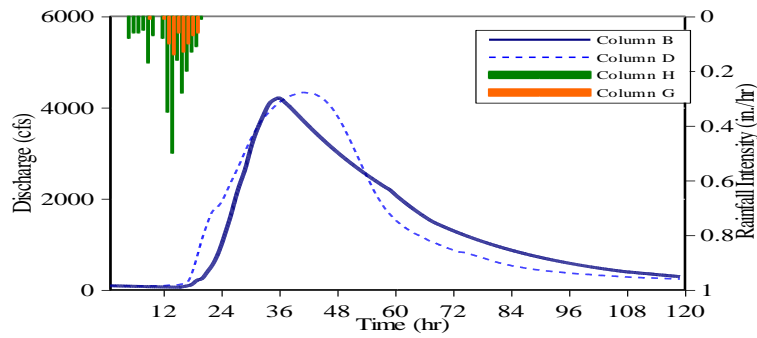| Storm #	Relative Error in Peak Flow (%)	Error in Time to Peak (hr)	Relative Error in Volume (%)
Whitewater River (07147070)			
1	3	5	16
2	4	1	24
Illinois River (07196500)			
3	13	9	23
4	4	7	6

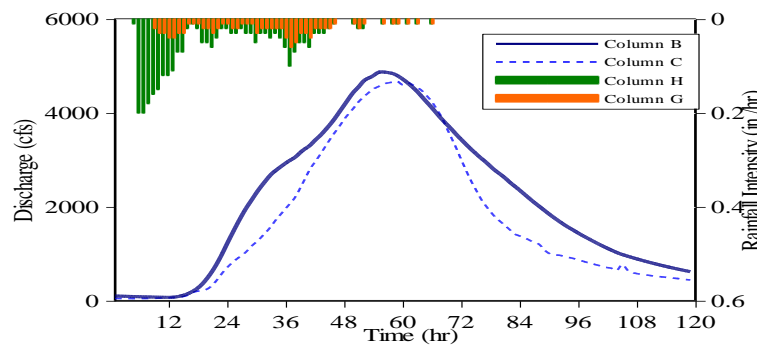**Figure 6 (a): Verification Result for Storm #1**

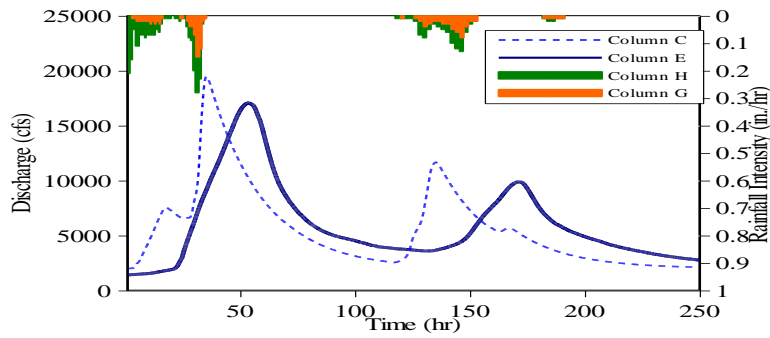**Figure 6 (b): Verification Result for Storm #2**

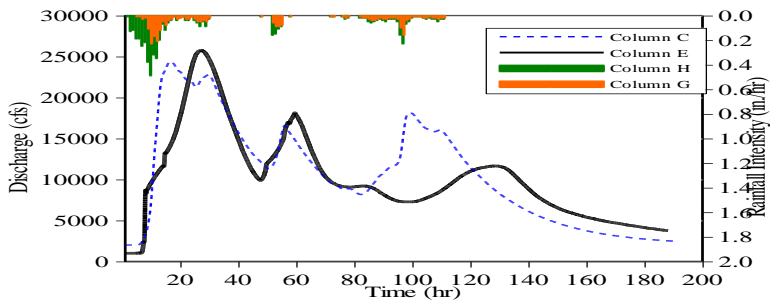**Figure 6 (c): Verification Result for Storm #3**

Figure 6 (d): Verification Result for Storm #4

For storm #1 on the Whitewater River watershed, the model did good job in simulating the peak flow; the relative error is 3%. The simulated hydrograph also matches the observed hydrograph well in the shape, but the simulated peak occurs 5 hours early. For storm #2 on the Whitewater River watershed, the simulated hydrograph has good agreement with the observed hydrograph. The relative error in peak flow is 4% and the error in time to peak is one hour. For storm #3 on the Illinois River watershed, the simulated peak occurs 9 hours too early, but the shape of hydrograph and the peak discharge are satisfactory. For storm #4, the model did good job in simulating the hydrograph shape and peak flow. The main peak occurs about 7 hours early. Overall, these simulation results are satisfactory.

In order to make use of all the information, times of concentrations and storage coefficients for the Whitewater River watershed and the Illinois River watershed were calibrated. The results are listed in Table 11:

Table 11: Calibration Results

Station Name	T_C (hr)	T_R (hr)
Whitewater River	25	31
Illinois River	39	22

Parameters from all of these eight watersheds are used to update the C value in the velocity equation and Eq. (32). The resulting C value is still 10 and the equation for storage coefficient is:

$$T_R = 0.0042 \frac{A^{1.38}}{S^{0.94}} \tag{33}$$

This equation has a correlation of determination of this equation is 0.98 and the standard error is 0.15.

VIII.   Summary and Discussion

The research presented in this paper improved the ModClark model by adding two features: a spatially distributed loss model and a spatially distributed velocity field. A new formula to calculate the spatially distributed velocity field was derived. The improved ModClark model was applied to 25 storm events on six watersheds. The time of concentration and storage coefficient were calibrated for each event. The calibration results are excellent. Based on the calibration results, two equations to estimate the time of concentration and storage coefficient were developed. This model and two parameters equations were applied to two ungaged watersheds to simulate four storm events. The results are satisfactory.

The ModClark model is a very promising model. Its use of radar-based precipitation for runoff estimation provides a major improvement for the modeling of spatially varied rainfall events.
The improved ModClark model retains the advantages of the ModClark model while adding the spatially distributed runoff curve numbers and velocity fields to account for the spatial variability in runoff generation and routing. It is an improvement over traditional unit hydrographs because it accounts for spatial patterns of rainfall, topographic characteristics, soils and land cover.

The improved ModClark model is conceptually and mathematically simple. It uses freely available radar-based precipitation data and digital morphology data sets to represent the spatial distribution of precipitation and hydrologic parameters in a watershed. This model and the two equations for the two global parameters provide a framework for simulating runoff hydrographs for watersheds. It has proven to be a very promising model.

The two parameter-forecasting equations (Eq. (28) and Eq. (33)) were calibrated from eight watersheds. These equations could probably be improved with additional calibration. Further calibrations of T_C and T_R on more gaged watersheds are needed.

The improved ModClark model does not account for the spatial variability of storage characteristics. Storage effects are modeled with a single hypothetical reservoir at the watershed outlet. If spatially variable storage effects were incorporated in the ModClark model, its prediction ability could be improved. Further research is needed to develop a method to account for the spatial variability of storage coefficients.

Further research is also needed to develop regional parameter-predicting equations. This study focused on the north-central partion of the Arkansas-Red River Basin. Since NEXRAD rainfall estimates and digital elevation model are available for the whole United States, this model could be applied to other watersheds in other regions. Watersheds with a variety of soils, land cover, and precipitation characteristics should be tested. Based on the calibration results from these gaged watersheds, an attempt should be made to develop regional parameter-predicting equations for use on ungaged watersheds.

References

[1] Ajward, M. H. (1996). A Spatially Distributed Unit Hydrograph Model Using A Geographical Information System. Ph.D. Dissertation. Civil Engineering Dept., University of Calgary, Canada.

[2] Barnes H. H. (1967). Roughness Characteristics of Natural Channels. Geological Survey Water-Supply Paper, Washington, D.C..

[3] Clark, C. O. (1945). Storage and the Unit Hydrograph. Transactions / American Geophysical Union, 110, pp. 1419-1446.

[4] Clement, R. W. (1987). Floods in Kansas and Techniques for Estimating Their Magnitude and Frequency on Unregulated Streams. Water Resources Investigations Report 87-4008, U.S. Geological Survey.

[5] ESRI (1997). Cell-Based Modeling with Grid 6.1: Supplement – Hydrologic and Distance Modeling Tools. Environmental Systems Research Institute, Redlands, CA.

[6] Hydrologic Engineering Center (1990). HEC-1 Flood Hydrograph Package User's Manual. Hydrologic Engineering Center, U.S. Army Corps of Engineers, Davis, CA.

[7] Hydrologic Engineering Center (1995). HEC-DSS User's Guide and Utility Program Manuals. Hydrologic Engineering Center, U.S. Army Corps of Engineers, Davis, CA.

[8] Hydrologic Engineering Center (1995). GridUtl User's Manual. Hydrologic Engineering Center, U.S. Army Corps of Engineers, Davis, CA.

[9] Hydrologic Engineering Center (1996). GridParm – Procedures for Deriving Grid Cell Parameters for the MoDClark Rainfall – Runoff Model – User's Manual. Hydrologic Engineering Center, U.S. Army Corps of Engineers, Davis, CA.

[10] Hydrologic Engineering Center (2000). HEC-HMS Hydrologic Modeling System. Hydrologic Engineering Center, U.S. Army Corps of Engineers, Davis, CA.

[11] Jenson, S. K. and Domingue, J. O. (1988). Extracting Topographic Structure From Digital Elevation Data for Geographic Information System Analysis. Photogrammetric Engineering and Remote Sensing, 54 (11), pp. 1593-1600.

[12] Johnstone, D., and Cross W. P. (1949). Elements of Applied Hydrology. Ronald Press Company, New York, NY.

[13] Kansas Water Resources Board (1967). Kansas Water Atlas.

[14] Kull, D. W. and Feldman, A. D. (1998). "Evolution of Clark's Unit Graph Method to Spatially Distributed Runoff." Journal of Hydrological Engineering, 3(1), pp. 9-18.

[15] Leopold, L. G., Wolman, M. G., and Miller, J. P. (1964). Fluvial Processes in Geomorphology. W. H. Freeman Inc, San Francisco.

[16] Maidment, D. R. (1992), Grid-Based Computation of Runoff: A Preliminary Assessment. Hydrologic Engineering Center, US Army Corps of Engineers, Davis, CA.

[17] Maidment, D. R. and Olivera, F. (1996). Unit Hydrograph Derived from A Spatially Distributed Velocity Field. Hydrological Processes, 10, pp. 831-844.

[18] McEnroe, B. M. (1992). Evaluation and Updating of Hydrologic Analysis Procedures Used by KDOT. Report No. K-TRAN: KU-92-1, Vol. I, Kansas Department of Transportation.

[19] National Resources Conservation Services (1976). Urban Hydrology for Small Watersheds, Technical Release No. 55, U.S. Department of Agriculture.

[20] National Resources Conservation Services (1972). National Engineering Handbook. Sec. 4, Hydrology.

[21] Peters, J. C. and Easton, D. J. (1996). Runoff Simulation Using Radar Rainfall Data. Water Resources Bulletin, 32(4), pp. 753-760.

[22] Pilgrim, D. H., (1976) Travel Times and Nonlinearity of Flood Runoff from Tracer Measurements on a Small Watershed. Water Resources Research, 12 (3), pp. 487-496.

[23] Rasmussen, P. P. and Perry, C. A. (2000). Estimation of Peak Streamflows for Unregulated Rural Streams in Kansas, Water-Resources Investigations Report 00-4079. U.S. Geological Survey.

[24] Unidata Program Center (1991). NetCDF User's Guide, An Interface for Data Access, Version 2.0, Boulder, Colorado.

[25] U.S. Geological Survey (1993), Digital Elevation Models, Virginia.

[26] Young, C. B., McEnroe, B. M., and Quinn, R. J. (1999). Utilization of Precipitation Estimates Derived from Composite Radar. Report No. K-TRAN: KU-96-7, Kansas Department of Transportation, 1999.

[27] Young, C. B. (1996). Use of Radar-Based Precipitation Estimations and Other Geospatial Data Sets in ARC/INFO GIS for Hydrologic Studies. M.S. Thesis, Civil Engineering Dept., the University of Kansas.